Basis Swap
A basis swap is a swap that involves the exchange of two floating rates, where the floating rate payments are referenced to different bases.

Both legs of a basis swap are floating but derived from different index rates (e.g. LIBOR 1-month vs 3-month).

Basis swaps could involve many different kinds of reference rates for the floating payments, such as 3-month LIBOR, 1-month LIBOR, 6-month LIBOR, prime rate, etc.

Basis swaps are settled in the form of periodic floating interest rate payments.
Basis Swap

Basis Swap Introduction (Cont.)

- A basis swap can be used to limit interest rate risk that a firm faces as a result of having different lending and borrowing rates.
- Basis swaps help investors to mitigate basis risk that is a type of risk associated with imperfect hedging.
- Firms also utilize basis swaps to hedge the divergence of different rates.
- Basis swaps are quoted as a spread over a reference index. For example, 3-month LIBOR is frequently used as a reference. Spreads are quoted over it.
From the leg 1 receiver perspective, the payoff of a basis swap or basis swaplet at payment date $T$ is given by

$$Payff_{receiver} = N\tau((R_1 - R_2)$$

where

- $N$ - the notional;
- $\tau$ - accrual period in years (e.g., a 3 month period $\approx 3/12 = 0.25$ years);
- $R_1$ - the floating rate of leg 1 in simply compounding;
- $R_2$ - the floating rate of leg 2 in simply compounding.

For payer of leg 1, the payoff of a swap or swaplet at payment date $T$ is given by

$$Payff_{payer} = N\tau((R_2 - R_1)$$
Valuation

- The present value of leg 1 is given by
  \[ PV_1(t) = N \sum_{i=1}^{n} (F_{1i} + s_1)\tau_i D_i \]
  where
  \( t \) is the valuation date and \( s_1 \) is the floating spread.
  \( D_i = D(t, T_i) \) is the discount factor.
  \( F_{1i} = \left( \frac{D_{i-1}}{D_i} - 1 \right) / \tau_i \) is the simply compounded forward rate.

- The present value of leg 2 is given by
  \[ PV_2(t) = N \sum_{i=1}^{n} (F_{2i} + s_2)\tau_i D_i \]

- The present value of an interest rate swap can be expressed as
  - From the leg 1 receiver perspective, \( PV = PV_1 - PV_2 \)
  - From the leg 1 payer perspective, \( PV = PV_2 - PV_1 \)
First of all, you need to generate accurate cash flows for each leg. The cash flow generation is based on the start time, end time and payment frequency of the leg, plus calendar (holidays), business convention (e.g., modified following, following, etc.) and whether sticky month end.

We assume that accrual periods are the same as reset periods and payment dates are the same as accrual end dates in the above formulas for brevity. But in fact, they are different due to different market conventions. For example, index periods can overlap each other but swap cash flows are not allowed to overlap.

The accrual period is calculated according to the start date and end date of a cash flow plus day count convention.
The forward rate should be computed based on the reset period (index reset date, index start date, index end date) that are determined by index definition, such as index tenor and convention. It is simply compounded.

The formula above doesn’t contain the last live reset cash flow whose reset date is less than valuation date but payment date is greater than valuation date. The reset value is

$$PV_{reset} = r_0N\tau_0D_0$$

where $r_0$ is the reset rate.
The present value of the reset cash flow should be added into the present value of the floating leg.

Some dealers take bid-offer spreads into account. In this case, one should use the bid curve constructed from bid quotes for forwarding and the offer curve built from offer quotes for discounting.

Sometimes there is a floating spread added on the top of the floating rate in the floating leg.
### Example

<table>
<thead>
<tr>
<th>Leg 1 Specification</th>
<th>Leg 2 Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Currency</strong></td>
<td>USD</td>
</tr>
<tr>
<td><strong>Day Count</strong></td>
<td>dcAct360</td>
</tr>
<tr>
<td><strong>Leg Type</strong></td>
<td>Float</td>
</tr>
<tr>
<td><strong>Notional</strong></td>
<td>10000000</td>
</tr>
<tr>
<td><strong>Pay Receive</strong></td>
<td>Receive</td>
</tr>
<tr>
<td><strong>Payment Frequency</strong></td>
<td>6M</td>
</tr>
<tr>
<td><strong>Start Date</strong></td>
<td>7/16/2015</td>
</tr>
<tr>
<td><strong>End Date</strong></td>
<td>7/16/2020</td>
</tr>
<tr>
<td><strong>Spread</strong></td>
<td>0.0020625</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Index Specification</th>
<th>Index Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Index Type</strong></td>
<td>LIBOR</td>
</tr>
<tr>
<td><strong>Index Tenor</strong></td>
<td>3M</td>
</tr>
<tr>
<td><strong>Index Day Count</strong></td>
<td>dcAct360</td>
</tr>
</tbody>
</table>
Reference:
https://finpricing.com/lib/EqAsian.html